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REVIEW

Introduction to Singular Perturbations. By R. E. O'MALLEY, JR. Academic Press, 1974. 206 pp. $ 16.50.

A work of applied analysis, this book concentrates on developing proven mathematical methods for the asymptotic solution of boundary-layer problems involving ordinary differential equations. There are preliminary expositions of regular perturbation theory and of various types of singular perturbation method.

Discussions of linear and nonlinear boundary-value problems follow, separated by a treatment of nonlinear initial-value problems. The book concludes with chapters entitled "The singularly perturbed linear state regulator problem", "Boundary value problems with multiple solutions arising in chemical reactor theory" and "Some turning point problems".

Those who are familiar with the author's extensive research in singular pertubation methods will recognize that material has been selected in conformity with his interests. This is certainly no deficiency, for some selection must be made from the large number of possible topics.

The essence of the author's approach to most problems is as follows. Consider a regular expansion in powers of a small parameter ϵ and require it to satisfy the equation and suitably selected boundary conditions. Assume that the full solution is the sum of the regular expansion and an expression that takes account of expected rapid variation in some region or regions. (In many cases, this expression will be a power series in ϵ with coefficients that are functions of a 'boundary-layer variable' such as x/ϵ. In other instances it is of a WBKJ form: a rapidly varying exponential times a power series in ϵ.) Having obtained a formal solution, one can show that the remainder satisfies a Volterra integral equation. This equation can be handled by estimating the difference between successive approximants. The asymptotic character of the formal solution is thereby established.

Comparison with Van Dyke's *Perturbation Methods in Fluid Mechanics* is natural, for Van Dyke's volume, which appeared exactly a decade ago, is in the same *Applied Mathematics and Mechanics Series*. Two-thirds of O'Malley's references are to papers dated within the last ten years, and much of his book reflects this recent work. Van Dyke's book is explicitly concerned with problems of fluid mechanics, which is an advantage to readers of this *Journal* but which makes Van Dyke's general message difficult to obtain for others. O'Malley gives or outlines proofs for most of his results but supplies explicit motivation for his assumptions only towards the end of his book, which makes for more difficult reading. Van Dyke's free expression of his considerable intuition permits him to range far beyond the limits of rigour. Van Dyke matches inner and outer expansions; O'Malley deals with uniform expansions. Both books have been used for courses, through only Van Dyke's contains problems for the student.

I should think that fluid dynamicists would find it advantageous to introduce material from O'Malley's book into their teaching *after* they have developed some formal ability using other texts, for although many of the problems considered are relevant to fluid mechanics, this is not explicitly demonstrated. Whatever their area of specialization, applied mathematicians will be glad that the fundamentals of singular perturbation theory have been given such a clear and wide-ranging introduction.

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